Homage to James Clerk Maxwell

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As one privileged to learn the fundamentals of electromagnetic theory at the feet of Nicholas Kemmer in the Old Tait Institute in Edinburgh during the1950's, it was inevitable that I should at an early stage acquire a deep admiration, indeed awe, in relation to the massive achievements of James Clerk Maxwell. The distillation of the great mass of experimental knowledge accumulated during the earlier decades of the Nineteenth Century into what we now know as Maxwell's equations provides evidence of real genius; the beauty, and ultimate simplicity, of these equations is still a matter of wonder for students of applied mathematics in our universities, and recognition of the significance of Maxwell's equations must count as one of the high points of any degree course in mathematics and physics. Maxwell's recognition of the electromagnetic nature of light paved the way for Einstein's theory of special relativity, as Einstein himself was the first to recognise. It seems to me that Maxwell merits a similar place in the great halls of scientific achievements of the latter half of the Nineteenth Century that Einstein later attained during the first half of the Twentieth Century. The name of James Clerk Maxwell should resonate, along with that of Isaac Newton, in all the schools of this country, in recognition of his truly marvellous achievements. His was, after all, the first unified field theory, a unification with far-reaching consequences, of the previously separate and somewhat haphazard sciences of electricity and magnetism.

Many years of university and college teaching in Cambridge have made me ever-more conscious of the great heritage of Maxwell. His portrait hangs, opposite that of Newton, in the Fellows' Parlour in Trinity College, and I always take pleasure in leading my guests to this portrait. It has been proposed that the portrait be moved to a more prominent position in the Dining Hall where undergraduates and graduate students of the College could also take inspiration from it; but such radical proposals take several decades to come to fruition in such a College!

When the Department of Applied Mathematics and Theoretical Physics (DAMTP) in Cambridge moved in 1964 from the Old Cavendish site to the Old Press site on Silver Street, it took with it some of the furnishings of the Old Cavendish which had been assigned to the Department. Among these was a very large Victorian desk, alleged to have been that of Maxwell. This desk took pride of place in the office of the Head of Department at DAMTP, an office that I occupied from 1983 to 1991. The desk is still in DAMTP, although no longer in the same office.

There was also a very large electromagnet, weighing about a ton, which found its way from the Old Cavendish into the basement of DAMTP. This massive piece of equipment undoubtedly dated from the late Nineteenth Century; I was convinced it must have been Maxwell's, but, although I consulted all the experts in the Cavendish, and others, we were unable to trace its pedigree back to Maxwell, and eventually alas it had to be disposed of in order to make way for more modern equipment.

Much of my own research has been concerned with the origin of magnetic fields in planets and stars, the subject known in the magneto-hydrodynamic context as 'dynamo theory'. It is instructive to look at what Maxwell had to say about this problem in Chapter 8 of Part 3 of his great *Treatise on Electricity and Magnetism*. Having discussed possible reasons for the secular changes of the Earth's magnetism (i.e. those changes on a timescale of the order of 40 years, often described as westward drift) Maxwell writes as follows:-

> *"What cause, whether exterior to the Earth or in its inner depths, produces such enormous changes in the Earth's magnetism, that its magnetic poles move slowly from one part of the globe to another? When we consider that the intensity of the magnetisation of the great globe of the Earth is quite comparable with that which we produce with much difficulty in our steel magnets, these immense changes in so large a body force us to conclude that we are not yet acquainted with one of the most powerful agents in nature, the scene of whose activity lies in those inner depths of the Earth, to the knowledge of which we have so few means of access."*

One can almost hear Maxwell deliver these words at the end of a lecture on terrestrial magnetism; they convey his own sense of wonder at one of the great phenomena of nature, in a style that is poetic in its intensity.

It was in fact only one hundred years later that a true understanding of the origins of the Earth's magnetic field began to emerge. This is rather remarkable, given that the equations governing the electromagnetic aspect of the problem were already known to Maxwell; indeed, since the displacement current turns out to be quite negligible in the context of dynamo theory, the equations governing the phenomenon are often described as the *'pre-Maxwell's equations'*. The great difficulty of the problem lies in the fact that a large part of the Earth's interior is liquid (mainly a molten alloy of iron and nickel) and it is the non-linear interaction between the fluid dynamics of this medium and the electric currents and magnetic fields that it can support that have for so long resisted mathematical analysis. The great breakthrough came in the 1960's and 1970's, with the realisation that a key to the dynamo mechanism (i.e. the mechanism whereby the Earth's magnetic field is generated and maintained) is to be found in the *'helicity'* of the fluid motions in the liquid core. This *helicity* is induced by Coriolis forces associated with the Earth's rotation: these forces tend to make fluid particles follow helical paths right-handed in the northern hemisphere, and left-handed in the southern hemisphere. It is now well understood that a combination of *helicity* and differential rotation within the liquid core provide just the conditions that are most conducive to dynamo action.

Helicity is a concept in fluid mechanics that has a very natural link with Kelvin's circulation theorem, and its consequence that *"vortex lines move with the fluid"*. As Kelvin was well aware, this implied the conservation of any knotted structure in the vorticity field of an ideal (frictionless) fluid. It is now known that *helicity* is the conserved property that properly describes this idea of *"knottedness"* of vortex lines.

Maxwell was clearly intrigued by this sort of concept. In Article 421 of his treatise, he discusses the Gauss invariant for two linked curves (which is related to *helicity*) and shows by a sketch that *"It is possible for two curves to be intertwined alternately in opposite directions, so that they are inseparably linked together though the value of the integral is zero"*. He goes on to say: *"It was the discovery by Gauss of this very integral, expressing the work done on a magnetic pole describing a closed curve in the presence of a closed electric current, and indicating the geometrical connection between the two closed curves, that led him to lament the small progress made in the geometry of position since the time of Leibnitz, Euler and Vandermonde. We have now however some progress to report, chiefly due to Riemann, Helmholtz, and Listing."* The *'geometry of position'* is what later became *'topology'*. It was of course during the same decade (the 1870's) that Tait, inspired by Kelvin, initiated his great work on the classification of knots.

In Maxwell's Paradoxical Ode *To Hermann Stoffkraft, Ph. D. (the Hero of a recent work called "Paradoxical Philosophy"[i])* one may find the following verse:-

  My soul's an amphicheiral knot
  Upon a liquid vortex wrought
By Intellect, in the Unseen residing,
  And thine doth like a convict sit,
  With marlinspike untwisting it,
Only to find its knottiness abiding;

The invariance of *helicity* is embedded in this verse. Perhaps Maxwell really was a hundred years ahead of his time!

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[i] Paradoxical Philosopy by Peter Guthrie Tait and Balfour Stewart, 1878