newsletter

OF THE JAMES CLERK MAXWELL FOUNDATION

Issue No.7 *Autumn 2016*

ISSN 2058-7503 (Print)
ISSN 2058-7511 (Online)

Maxwell's Equations: the Tip of an Iceberg [1]

by Professor Peter Higgs, CH, FRS, FRSE – University of Edinburgh (Physics Nobel Laureate, 2013)

My title describes Maxwell's equations as the tip of an iceberg and I am going to try to tell you something about that iceberg.

In 1865, Maxwell's electromagnetic field equations were the first fully fledged *field theory* of forces. The term *field* had been introduced twenty years earlier by Faraday in a paper in which he referred to *magnetic fields*. Maxwell did what Faraday was not well equipped to do because of his lack of mathematics; he arrived at the fully fledged theory of electromagnetism. Not only was Maxwell's theory the first fully fledged field theory, it was actually (although Maxwell himself was not aware of this) the first relativistic field theory, in the current sense of the term.

Maxwell's thinking about space and time was very Newtonian – absolute time and all that – so what he thought of as the relation between inertial frames of reference (the *Galilean transformation*) led to the conclusion that his equations were valid only in the reference frame of what was called the *luminiferous ether*. It was Einstein, forty years later, who put that right by modifying Newton's equations (so as to be compatible with the *Lorentz transformation*) but keeping Maxwell's. That was one of the ways in which Maxwell was a pioneer.

(The second relativistic field theory was formulated by Einstein – his 1915 theory of gravitation – which he arrived at with the knowledge of what Maxwell had done before him. I will not have anything to say about gravity.)

Quantum Mechanics

Around 1925, sixty years after Maxwell's equations, quantum mechanics arrived. Soon Maxwell's theory was placed in a quantum mechanical framework by Dirac, who next formulated the relativistic quantum theory of electrons. The combination of these, known as *quantum electrodynamics* (QED) arrived around 1930.

After some years of difficulty in calculations of quantum effects – the problem of infinities – around 1950 the theory finally became a fully successful *quantum field theory*. In fact, it is still the most accurate theory that we have in its experimental predictions. That is what followed, in the twentieth century, from Maxwell's work.

I want to turn now to the subject of generalisations of Maxwell's equations and their relation to other forces which were not discovered until the twentieth century.

Noether's Theorem

First, I want to highlight an important step in our understanding of Maxwell's equations which arose from the theorem proved in 1915 by Emmy Noether. This was the theorem that proved the connection between symmetries and conservation laws in dynamics.

At the time, the known applications of Noether's Theorem were to space-time symmetries (Euclidean symmetry), whereby translation symmetry in space gives rise to conservation of linear momentum, translation symmetry in time to energy conservation, and rotational symmetry to conservation of angular momentum. But that list has omitted one well known conservation law; electric charge is conserved and this is built into Maxwell's equations. What does this have to do with symmetry?

Schrödinger's Equation

The answer to this question did not become clear until Schrödinger, in 1926, wrote down his wave equation for a charged particle in an electromagnetic field. It then became clear that a new kind of symmetry was involved, which had nothing to do with space and time. A Schrödinger wave function is a complex number, so two real numbers are involved; and you can think of them as defining a point in a two-dimensional *Argand diagram*. It transpired that the conservation of charge is the consequence, by Noether's theorem, of the symmetry of this wave function under rotations in the Argand plane. This is the germ of all the other symmetries which were encountered later.

Classical forces not the whole story

In the twentieth century it began to be clear that the classical forces, electromagnetism and gravity, were not the whole story. First, the phenomena of radioactivity, in particular what is called *beta-decay*, made it clear that there is what we now call a *weak nuclear force* of very short range – perhaps even a contact force – between the particles involved in this process.

What is more, when Rutherford and others started probing the structure of the atomic nucleus, it became clear that there must also be a *strong nuclear force* to hold the constituents of the atomic nucleus together. In 1932, Chadwick's discovery of the neutron showed that the constituents were protons and neutrons, particles which turned out to be rather similar – close in mass, with the same spin, differing by one carrying electric charge and the other carrying none: as far as the strong nuclear force was concerned they were similar.

Heisenberg and Ivanenko (1932) introduced an enlargement of the type of symmetry that underlies charge conservation, to a set of operations some of which mix the proton and the neutron wave functions; this symmetry became known as *isotopic spin symmetry*.

The subsequent development of *elementary particle physics* involved the discovery of still bigger *approximate symmetries* of this type.

→

1 Adapted from his address to the Royal Society of Edinburgh on 9 November 2015 on the occasion of the 150th anniversary of the 1865 publication of Maxwell Equations.

Theories of these new forces

When it came to theories of these new forces, the situation was not very good.

There was a theory of the strong nuclear force which was not too bad when the separation of the particles was large by nuclear standards and which incorporated the *isospin* symmetry. This was the work of Kemmer (1936) based on earlier work of Yukawa (1935); when this theory was pushed too far in order to understand the properties of protons and neutrons better it did not work very well – agreement with experiment was poor.

The situation with the weak nuclear force was even worse. It became clear that there were similarities between the weak and electromagnetic forces, which suggested that the weak nuclear force should be mediated by a field whose *quanta* [2] are particles of spin one like *photons*, the quanta of electromagnetism. These particles would have to be very massive (unlike the photon which is massless) because in quantum theory the mass of the quantum goes inversely as the range of the force.

But when theorists tried to handle the quantum field theory of a such a force, they got nonsense. It was 'not even wrong'; it was impossible to calculate with it because quantum effects gave rise to infinities which could not be *renormalised* away as had been done in QED. This was a disaster for the theory; many theorists gave up trying to use quantum field theory in elementary particle physics.

Maxwell's Equations generalised

Meanwhile, Maxwell's equations had been generalised so as to incorporate bigger symmetries; there had been a hint of this already, in 1938, in some work of the Swedish physicist Oscar Klein. The first full description of such a theory was given in 1954 by Yang and Mills: it was also in the unpublished PhD thesis of Ronald Shaw.

For the isospin symmetry of the strong force, it required three copies of a Maxwell field interacting with one another, which made it a highly non-linear theory.

The big problem was that it appeared to have massless quanta, corresponding to long-range forces. And so for many years it was put on one side as a non-starter.

Spontaneous Breaking of Symmetry

However six years later, in 1960, a new concept, *spontaneous* breaking of a symmetry, was imported into relativistic quantum field theory from the physics of condensed matter by the Japanese-born theorist Yoichiro Nambu. He had learned about the successful theory of superconductivity (Bardeen, Cooper and Schrieffer, 1957) from Schrieffer, and he saw that, when expressed in the language of (non-relativistic) quantum field theory, this theory had a ground state (the state of lowest energy) which was asymmetric with respect to the symmetry which underlies electric charge conservation. Now in relativistic quantum field theory, the ground state is what we know as the *vacuum* (absence of particles) [3] and, if it is asymmetric with respect to a symmetry, the states of higher energy (particles present) exhibit broken symmetry too. Nambu tried out this idea on the symmetries of the strong nuclear force and obtained interesting results; he generated masses for the protons and neutrons *from scratch* – they were absent from the underlying field equations but appeared in the quantum states.

But there was a big snag; the theory also predicted quantum states containing massless particles of spin zero. That was a disaster because if such particles existed in our Universe, experimentalists would have found them long ago – there is no threshold energy required for their creation – and they would contribute substantially to energy loss from stars, which we know is mostly due to electromagnetic radiation (photons).

Goldstone's Theorem

Nambu's theory seemed to be dead in the water, especially after Jeffrey Goldstone of Cambridge (using an alternative formulation of spontaneous symmetry breaking as a feature of classical field theories) pointed out that the difficulty was intuitively obvious and should be a general consequence of spontaneous symmetry breaking in relativistic theories. By 1962, a theorem had been proved by Salam and Weinberg (but known as the Goldstone Theorem because he agreed to co-author the paper) which seemed to make it inescapable.

Loophole in the Goldstone Theorem

But, two years, later it was discovered that there was a loophole in the theorem; the axioms did not apply to Maxwellian field theories. All that had to be done was to use Nambu or Goldstone symmetry breaking fields as sources of Maxwellian fields. Such a combination of theories gave just what was needed, theories in which some forces were short range and others long range, the corresponding quanta being massive and massless.

In 1967, Weinberg and also Salam, built on a Maxwellian theory by Glashow (1961), who had proposed a certain unification of weak and electromagnetic interactions, by using the Goldstone version of spontaneous symmetry breaking to generate masses for the quanta of the weak forces.

In 1973, this theory received experimental confirmation. Much of the impetus for the crucial experiment came from the work of two Dutch theorists, Veltman and 't Hooft, who by 1971 had proved that Maxwellian quantum field theories, including those with spontaneous symmetry breaking arising from a certain class of Goldstone sources (such as the Weinberg-Salam theory) were as viable and calculable as quantum electrodynamics.

Thus by the late twentieth century Maxwell's equations had been enlarged to produce what we now call the *electroweak theory* which contains four Maxwellian fields, one of which is electromagnetic.

Quantum Chromodynamics

By the mid nineteen seventies, more powerful methods for analysing quantum field theories – again developed previously for condensed matter physics – had shown that some Maxwellian theories had a quite unexpected behaviour; the fields could confine particles of spin one-half in many-particle composite objects which could not break up, the fields being confined too. In this way, a successful Maxwellian theory of the strong nuclear force, known as *quantum chromodynamics* (QCD) was developed. The confined particles are three *quarks* making up a composite proton and neutron [4]: eight Maxwellian fields (whose quanta are called *gluons*) are needed.

Conclusion

To conclude, we now know a total of twelve Maxwellian fields, four for the electroweak forces and eight for the strong forces. Quite an iceberg!

2 The concept of *quanta* of a field had been introduced by Einstein in 1905; it was what ultimately earned him his Nobel Prize in 1921.
3 The quantum vacuum is not as empty as physicists used to believe; it may contain uniform directionless fields!
4 By 1970 there was experimental evidence for this compositeness.

newsletter OF THE JAMES CLERK MAXWELL FOUNDATION

Maxwell and Control Theory

By Professor Rodolphe Sepulchre, BSc, PhD (Louvain), Professor of Control Engineering at the University of Cambridge

"We have decided to call the entire field of control and communication theory, whether in the machine or in the animal, by the name Cybernetics, which we form from the Greek χυβερνήτης or steersman. In choosing this term, we wish to recognize that the first significant paper on feed-back mechanisms is an article on governors, which was published by Clerk Maxwell in 1868, and that governor is derived from a Latin corruption of χυβερνήτης."

(Wiener Norbert, Cybernetics, or Controls and Communication in the Animal and the Machine – Cambridge, Mass.: M.I.T. Press, 1948)

Cybernetics

When the great American mathematician Norbert Wiener (1894 – 1964) published the above masterpiece on control and communication theory, he had to invent a suitable name and a suitable heritage which would delineate the new field (which was relevant to the science of the machine and to the kingdom of the animal) and which would also revolutionize 20th century engineering. He chose the name *cybernetics*, from the Greek translation of *governors*.

Maxwell's paper, *'On Governors'*, is perhaps one of the most unusual contributions by the father of electromagnetism. The paper was written over a few weeks while Clerk Maxwell lived at his home in Scotland, semi-retired from academic teaching. It is a mathematical paper about an engineering topic. It concludes several years of study of speed regulation mechanisms – quite a distraction from Maxwell's famous treaty on electromagnetism published in 1873.

Maxwell's paper was not a model of scientific writing, had no practical impact and illustrated technology that was soon to become obsolete. Maxwell's paper was virtually unknown until it received publicity from Wiener. Yet, it contains the roots of control theory to become to-day an important branch of engineering with an ever increasing range of applications. These range from aerospace and manufacturing to artificial intelligence and bio-inspired robotics.

Cruise control in the 18th and 19th Centuries

A *cruise control* mechanism is a device that automatically regulates the speed of a device.

The principle is the same as that for the feedback mechanism of a thermostat; namely that an error signal is derived by comparing the current speed (of the device) with the desired speed (thus deriving the *speed-error*). A corrective torque is then applied to the device so as to minimise the speed-error.

Today's cruise control systems are based on electronic sensors and a few lines of code. Although these resources were not available in Maxwell's day, speed regulation was already critical for the operation of many devices. *Governors* were the cruise control systems of Maxwell's time.

In the 18th century, the feedback mechanism was the result of clever mechanical links between the speed sensor and the motor. It had been pioneered for steam engines by James Watt (1736 – 1819) who connected a *fly-ball speed sensor* to the valve controlling the steam inflow in the engine (Figure 1).

During the 19th century, the design of governors became a sophisticated engineering art to which many renowned physicists of the time such as Airy, Foucault, Kelvin, and Gibbs all contributed.

Governors and feedback control

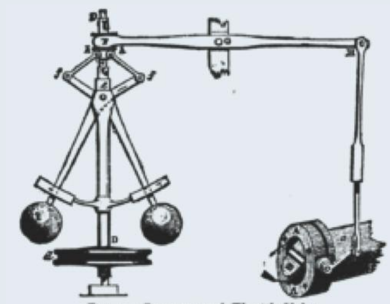

Figure 1: James Watt's centrifugal speed controller (courtesy of Wikipedia).

Clerk Maxwell was exposed to governors from 1861, when the British Association for the Advancement of Science asked him to be a member of a committee responsible for the establishment of electrical standards, particularly of resistance (the ohm). The committee conceived an experiment to measure the resistance of a circular coil of wire by spinning it rapidly about a vertical axis. The speed of the coil had to be closely regulated for the method to give an accurate measurement. For this purpose, the committee borrowed a governor designed by one of its members, Fleeming Jenkin (1833 – 1885), the Professor of Engineering at Edinburgh University.

The manuscripts of Maxwell (carefully scrutinised by the scholar A.T. Fuller during his term in the control group at Cambridge University) attest to the continuing interest of Maxwell in the intricacies of governors. This continuing interest took place over the seven years separating Maxwell's first encounter with the Jenkin's governor and the publication of his paper *'On Governors'*.

Maxwell was less interested in the mechanisms themselves than in their underlying principles. This required formulating abstract questions as to their mathematical structure, the generality of which would make them relevant beyond their particular rôle at the time.

It was Maxwell's attempt to turn the art of designing governors into the science of feedback that must have caused Wiener to designate the illustrious physicist as the father of control theory.

Stability analysis

Maxwell placed the stability question at the core of his analysis of governors by saying in his paper:

> *"There is a very general and very important problem in Dynamics... It is this: having found a particular solution... to determine whether a slight disturbance would cause a small periodic variation or a total derangement of the motion".*

→

The feedback interconnection of two (otherwise stable) systems might result in instability – a fundamental reason why feedback requires a mathematical theory.

Based on his earlier work on Saturn Rings, Maxwell attacked the stability question through the *linearisation* of the resulting equations of motion.

By using linearisation, Maxwell reduced stability analysis to the algebraic problem of determining whether all the roots of a certain polynomial have negative real part. This question is of central importance in Maxwell's paper and is repeatedly found in the mathematical problems he designed for students while teaching at King's College, London, in the early 1860's. In this regard, his 1868 paper, ends with a question:

> ***"I have not been able completely to determine these conditions for equations of a higher degree than the third; but I hope that the subject will obtain the attention of mathematicians".***

E.J. Routh (1831 – 1907), the famous Cambridge mathematical tutor and former classmate of Maxwell at Peterhouse College (where Maxwell spent his first term before migrating to Trinity College) became interested in this question. In 1877, he provided a general solution for this stability problem for which he won the 1877 Adams Prize, exactly 20 years after Maxwell had won the 1857 Adams Prize for his memoir on Saturn rings (Maxwell was on the Smith's Prize Committee which set the topic for the 1877 Adams Prize).

It is unclear whether Maxwell's paper *'On Governors'* would have been known to Wiener without the interest of Routh in solving the open question raised by Maxwell. The solution (now called the Routh-Hurwitz stability criterion) is still taught today in undergraduate control classes and stability analysis through linearisation is still the first step of any modern control design.

Proportional and Integral Feedback Control

Further to his stability analysis, Clerk Maxwell devoted a large part of his 1868 paper to highlight a fundamental distinction between *moderators* and *governors*.

In moderators, the correcting torque is proportional to the speed-error whereas, in *governors*, the correcting torque is proportional to the integral of the speed-error (*integral-action or integal-control*).

Using the above terminology, the fly-ball regulator (invented by James Watt for his steam engine) is a moderator as it uses only a rigid mechanical transmission from the fly-ball to the correcting valve.

In contrast, most of the designs of Maxwell's time, including the design of Fleeming Jenkin, were governors as they included integral-action in the design.

In his 1868 paper, Maxwell observed that governors needed an independent moving piece to produce a response proportional to the integral of the speed-error, a reason for the increasing sophistication of the design.

In the Jenkin's governor, this response was produced by an idler pulley. This pulley was connected through a worm gear to the brake-drum and to a counterweight suspended in a viscous fluid in a narrow upright cylinder (*centrifugal pendulum*) (Figures 2 and 3).

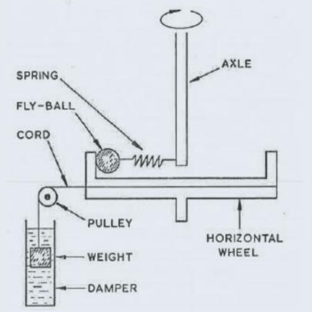

Figure 2: Principle of Fleeming Jenkin's flying ball governor, courtesy Transactions of ASME: Journal of Dynamical Systems, Measurement and Control, 1976.

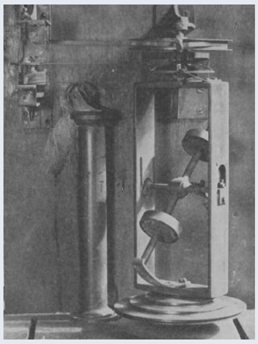

Figure 3: Fleeming Jenkin's governor, as used with Maxwell when determining the measurement of the ohm (Courtesy Whipple Science Museum, University of Cambridge).

A connection between the two systems (fly-ball and brake-drum) was established whenever the speed of the main system exceeded the desired value. Then, the excessive centrifugal force pressed the fly-ball against the brake-drum which moved. The integral response mechanism was thus set in motion. The braking torque on the main shaft would continue to increase until the centrifugal pendulum had released the idler pulley, that is, until the speed-error had entirely disappeared. The magnitude of the braking torque was proportional to the time integral of the speed-error.

Maxwell's paper explained why minimising the time integral of the speed-error was necessary for achieving exact speed regulation (i.e. a zero steady-state error). It clarified why the minimisation of the time integral of the speed-error was not incompatible with stability, an issue that appeared unclear at the time (and continues, even today, to take many students by surprise!).

The ability of a feedback system to achieve exact regulation, in spite of uncertainty, is a fundamental principle of modern control theory. Integral control is the simplest manifestation of this principle. Millions of regulators deployed in today's industrial world are in operation just for their integral action, because it enables perfect adaptation i.e. exact regulation in uncertain and changing environments. Integral action has also been identified in many biological regulatory feedback loops.

Maxwell's heritage

For many, it comes as a surprise that Clerk Maxwell concerned himself with an engineering-type question at the very time of producing his fundamental equations (Maxwell's Equations) of electromagnetism. However, Wiener recognised that it must have taken one of the greatest of scientific minds to foresee the importance of feedback and to call for a mathematical science of feedback.

One hundred fifty years later, control theory is still a young discipline. Stability analysis, through linearisation and integral-control, are part of its fundamentals. Most importantly, control engineers, neuroscientists and roboticists agree that feedback remains one of the most important concepts of contemporary science.