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New electromagnetic surface waves: Voigt surface waves

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Conventional electromagnetic surface waves

The simplest (nontrivial) solutions to the Maxwell equations are uniform plane waves, familiar to all students of electromagnetic theory. In the simplest case, these propagate in a linear homogeneous material that extends to infinity in all directions.

Surface waves are quite different. These are localised waves that straddle the planar interface of two dissimilar partnering materials[1]. The energy density of a surface wave far away from the interface is negligibly small. If the interface is removed by making the two partnering materials identical, the surface wave disappears. In addition, there is no guarantee that a selected pair of partnering materials will support the existence of a surface wave – the parameters specifying the electromagnetic characteristics of both partnering materials may be required to satisfy certain constraints in order for a surface wave to exist. Thus, surface waves are much more delicate entities than plane waves.

Surface-plasmon-polariton waves

Since the beginning of the twentieth century, several different types of electromagnetic surface wave have been identified. The type is determined by the electromagnetic characteristics of the two partnering materials. The most widely studied type is the surface-plasmon-polariton (SPP) wave which is guided by the interface of an insulator and a metal. While SPP waves cannot be excited by direct illumination, their excitation is readily achieved indirectly via coupling with a prism or surface-relief grating, for examples. SPP waves are of major technological importance: they have been widely exploited for sensing chemical and biochemical substances.

Another major area of application is in microscopy; and further applications in optical communications and solar energy harvesting are on the horizon. As metals absorb electromagnetic energy, SPP waves travel only relatively short distances.

Dyakonov waves

Another type of surface wave – well established both theoretically and experimentally – is the Dyakonov surface wave, which propagates at the interface of two insulators. At least one of the partnering materials must have electromagnetic characteristics that vary with the direction of propagation – such materials are called anisotropic. Unlike other types of surface wave, such as SPP waves, Dyakonov surface waves can propagate over quite large distances; accordingly, they represent attractive propositions for applications involving long-range optical communications, for example.

Voigt surface waves

Our understanding of electromagnetic surface waves took a step forward recently when a fundamentally new type of surface wave, known as a Dyakonov–Voigt (DV) surface wave[2], emerged as the solution to a canonical boundary-value problem involving the interface of two insulators. One of the partnering materials is isotropic while the other is anisotropic. These DV surface waves are similar to Dyakonov surface waves insofar as they are guided by the interface of two insulators, but there are crucial differences: (i) The fields of DV surface waves decay as the product of a linear and an exponential function of distance from the interface in the anisotropic insulator, whereas the fields of Dyakonov surface waves decay only exponentially with distance from the interface in the anisotropic insulator. (ii) DV surface waves propagate only in four distinct directions in the interface plane whereas Dyakonov surface waves propagate for four continuous ranges of directions.

In a related study, another new type of surface wave, namely a surface-plasmon-polariton-Voigt (SPPV) wave[3], recently emerged as the solution to a canonical boundary-value problem involving the interface of an anisotropic insulator and a metal. The relationship between SPPV waves and SPP waves is analogous to the relationship between DV surface waves and Dyakonov surface waves.

1 J.A. Polo Jr., T.G. Mackay, A. Lakhtakia, Electromagnetic Surface Waves: A Modern Perspective (Elsevier, 2013).
2 T.G. Mackay, C. Zhou, A. Lakhtakia, Proc. R. Soc. Lond. A. 475, 20190317 (2019).
3 C. Zhou, T.G. Mackay, A. Lakhtakia, Phys. Rev. A 100, 033809 (2019).

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Theory underpinning electromagnetic surface waves

The properties of conventional and Voigt surface waves are most easily appreciated by considering the planar interface $z = 0$ of two unbounded regions: $z > 0$ filled with material *A* and $z < 0$ filled with material *B*. Suppose that material *A* is anisotropic while material *B* is isotropic. For DV surface waves both partnering materials are insulators whereas for SPPV waves one of the partnering materials is an insulator and the other is a metal. The propagation of surface waves is governed by the Maxwell equations. In fact, only the two Maxwell equations involving the curl operator are needed.

These Maxwell equations are conveniently recast as a matrix of ordinary differential equations

$$\frac{d}{dz}[\mathbf{f}(z)] = \begin{cases} \left[\underline{\underline{P}}_A\right] \cdot [\mathbf{f}(z)]\,, & z > 0 \\ \left[\underline{\underline{P}}_B\right] \cdot [\mathbf{f}(z)]\,, & z < 0 \end{cases}, \qquad (1)$$

where the vector $[\mathrm{f}(z)]$ has 4 components: the amplitudes of the *x* and *y* components of the electric field and the amplitudes of the *x* and *y* components of the magnetic field. The 4 x 4 matrices $[\underline{\underline{P}}_A]$ and $[\underline{\underline{P}}_B]$ are specified by the electromagnetic characteristics of materials *A* and *B*, respectively.

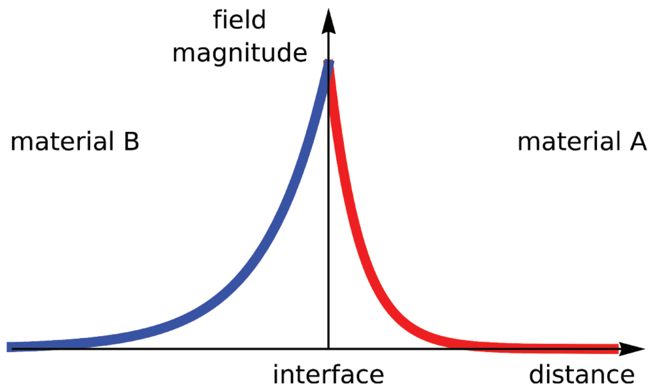
Let us consider the simplest case. For conventional surface waves, like SPP and Dyakonov surface waves, the matrix $[\underline{\underline{P}}_A]$ has four eigenvalues and four linearly independent eigenvectors. Consequently, the field amplitudes of the surface waves decay exponentially as *z* increases in the region $z > 0$.

For Voigt surface waves, the matrix $[\underline{\underline{P}}_A]$ has fewer than four eigenvalues and fewer than four linearly independent eigenvectors. Consequently, the decay of surface-wave field amplitudes as *z* increases in the region $z > 0$ is governed by the product of a linear function of *z* and an exponential function of *z*.

For both conventional and Voigt surface waves, the matrix $[\underline{\underline{P}}_B]$ has four eigenvalues and four linearly independent eigenvectors; and accordingly the surface-wave field amplitudes decay exponentially as *z* decreases in the region $z < 0$.

Schematic representations

Schematic representations of conventional and Voigt surface waves are provided in Fig. 1. For the conventional surface wave represented in Fig. 1 (top), the surface wave is tightly localised at the interface and the field magnitude decays exponentially in both directions as distance from the interface increases. Generally, the rate of decay in material *A* is different to the rate of decay in material *B*. In contrast, for the Voigt surface wave represented in Fig. 1 (bottom), the surface-wave field magnitude decays exponentially in material *B* but the form of decay in material *A* is quite different as it is specified by a linear-exponential function of distance from the interface. Accordingly, the energy density of a Voigt surface wave is not concentrated exactly at the interface but at a some distance – which may be short or considerable depending on the electromagnetic characteristics of materials *A* and *B* – from the interface in the $z > 0$ region. That is, Voigt surface waves are localised to a neighbourhood of the interface, but their localisation is quite different to that of conventional surface waves, because of their differently distributed energy densities.



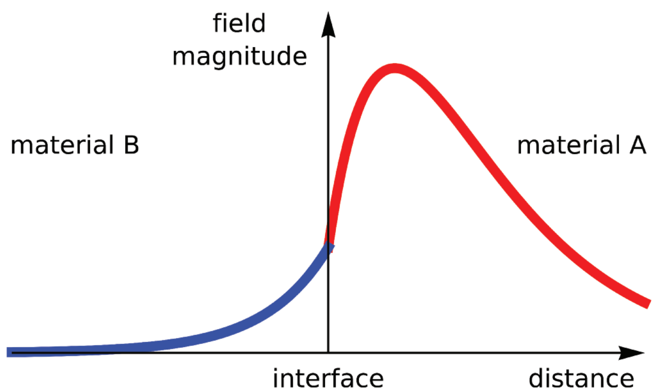


Figure 1: Schematic representations of a conventional surface wave (top) and a Voigt surface wave (bottom). Field magnitudes are graphed against distance from the interface. The decay for the conventional surface wave is exponential in materials *A* and *B*; and the decay for the Voigt surface wave is exponential in material *B* but linear-exponential in material *A*.

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Voigt plane waves

There are certain features shared by Voigt surface waves, namely DV surface waves and SPPV waves, and a special type of plane wave called a Voigt plane wave, named after the German physicist Woldemar Voigt (See Fig. 2). As for Voigt surface waves, anisotropy is essential for the existence of Voigt plane waves. These plane waves propagate in certain dissipative anisotropic insulators[4]. In general, two plane waves propagate in a given direction in such materials, with each plane wave being associated with a distinct wavenumber and eigenvector. Each plane wave decays exponentially in the direction of propagation.

But there exist special directions of propagation – distinct from the directions aligned with the optic axes – along which both plane waves have only one wavenumber and their eigenvectors are not distinct from each other either. For these propagation directions, the two plane waves coalesce to form a Voigt plane wave. Furthermore, the decay of Voigt plane waves in the direction of propagation is not simply exponential; instead their decay is governed by the product of a linear function and an exponential function of propagation distance. Voigt plane waves were first investigated experimentally and theoretically for pleochroic crystals (i.e. crystals that appear to be of different colours when viewed from different directions) but in recent times interest has grown in engineered materials that permit Voigt plane-wave propagation.

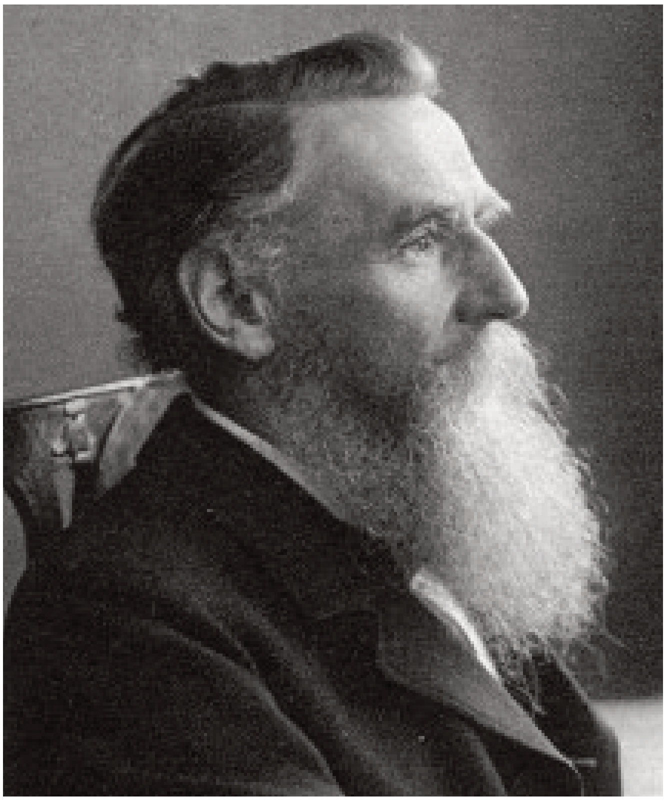

Figure 2: Woldemar Voigt (born Leipzig 1850, died Göttingen 1919), head of Mathematical Physics Department at the Georg August University of Göttingen, renowned for his work on crystal physics and electro-optics. Photograph courtesy of Emilio Segrè Visual Archives, American Institute of Physics.

Why are Voigt surface waves particularly interesting?

Plane waves, spherical waves and surface waves, arising as solutions to the Maxwell equations, are the well-established basic building blocks for much of applied electromagnetic theory.

Fundamentally new types of wave solution to the Maxwell equations are exceedingly rare. Therefore, the recent emergence of DV and SPPV surface waves, with their unique localisation characteristics, marks a significant milestone.

As well as their importance for fundamental research, Voigt surface waves appear promising for technological applications. Potentially, Voigt surface waves offer an additional operational mode for applications in optical sensing and applications in optical communications, for examples. Moreover, owing to their unique localisation, the additional mode offered by Voigt surface waves is generally not in the same spatial location as the conventional surface-wave mode. Thus, the prospect of parallelizing applications of surface waves emerges. For examples, (a) in optical communications, a signal '0' could be transmitted by a conventional surface wave while a signal '1' could be transmitted by a Voigt surface wave; and (b) in optical sensing, the presence of a substance-to-be-sensed could be signalled via a Voigt surface wave while the presence of a different substance-to-be-sensed, at a nearby but different location, could be signalled via a conventional surface wave.

Further research

However, further research on the excitation of Voigt surface waves in practical settings, such as involving prism- or grating-coupled configurations with realistic materials, is required before applications can be pursued in earnest.

4 W. Voigt, Phil. Mag. 4, 90–97 (1902).

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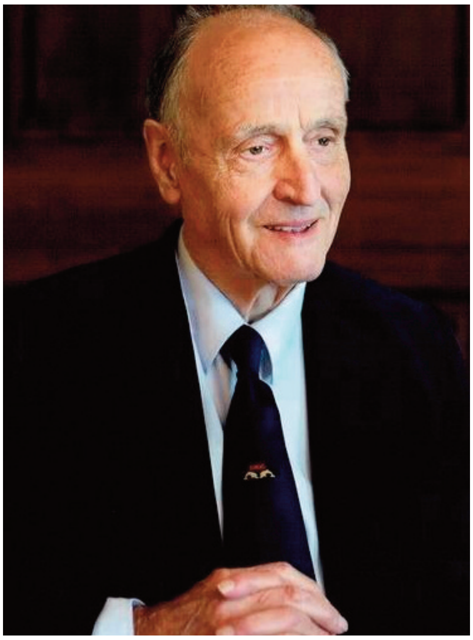
Tribute to Professor David S. Ritchie, MA FRMetS FRSE *(1923–2020)*

By Trustees of the Clerk Maxwell Foundation



Till, in the twilight of the gods,
When earth and sun are frozen clods,
When, all its energy degraded,
Matter to aether shall have faded,
We, that is, all the work we've done,
As waves in aether, shall for ever run,
In ever-widening spheres though heavens beyond the sun.

Extract from the poem 'A Paradoxical Ode'
by James Clerk Maxwell.

The Clerk Maxwell Foundation was formed in 1977 to honour and promote the memory and scientific contributions of James Clerk Maxwell, Scotland's most eminent physicist.

In 1987, David Ritchie was approached by our founder, Sidney Ross, to become a Trustee of the Clerk Maxwell Foundation. He was a trustee of the Foundation for thirty-two years and for part of this period he was our Chairman and later our Honorary President. The portrait of David (on the right) now hangs in the Foundation's rooms. David passed away peacefully in September of this year.

David Ritchie was educated at the Edinburgh Academy and at Cambridge University, as was Clerk Maxwell. David studied engineering. After serving in World War II, he joined, in 1948, the Glasgow company Bar and Stroud becoming, in due course, its Research Director. Although his primary work was with submarine periscopes and their ability to exploit all parts of the electromagnetic spectrum, he was also involved with the development of rangefinders and night-sights. After he retired from Barr and Stroud in 1986, David became a governor at Paisley University and Visiting Professor in Management Technology Innovation at Strathclyde University.

In 1993, David, as the Director of Development of the Maxwell Foundation, took on, with the assistance of other trustees, the task of raising £500,000 to enable the Foundation to buy the town house in the New Town of Edinburgh (14 India Street) where James Clerk Maxwell was born. David gave a significant personal gift to help the Foundation to buy the house. Furthermore, David and Professor Elmer Rees secured a substantial interest free loan from the Scottish Office for the same purpose. He worked tirelessly in writing many letters to individuals, trusts and charities to seek contributions and, with assistance from other trustees, succeeded in raising the necessary funds to buy 14 India Street and thereby firmly established Maxwell's legacy in Edinburgh.

Buying the house, where Maxwell was born, was acknowledged by the Founder of the Maxwell Foudation, Sydney Ross, as being a 'master-stroke'. Sydney Ross acknowledged that, without David working to help to raise the money to buy the house, the Foundation would only have ever existed on paper. The purchase of the house has had a transformative effect on the Foundation.

David Richie was elected a Fellow of the Royal Society of Edinburgh in 1997. He donated to the Royal Society of Edinburgh a fine portrait of Clerk Maxwell which now hangs in the Society's 'Maxwell Room' along with a hologram of the Maxwell statue which is located near the Society's premises.

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