Saturn's Rings

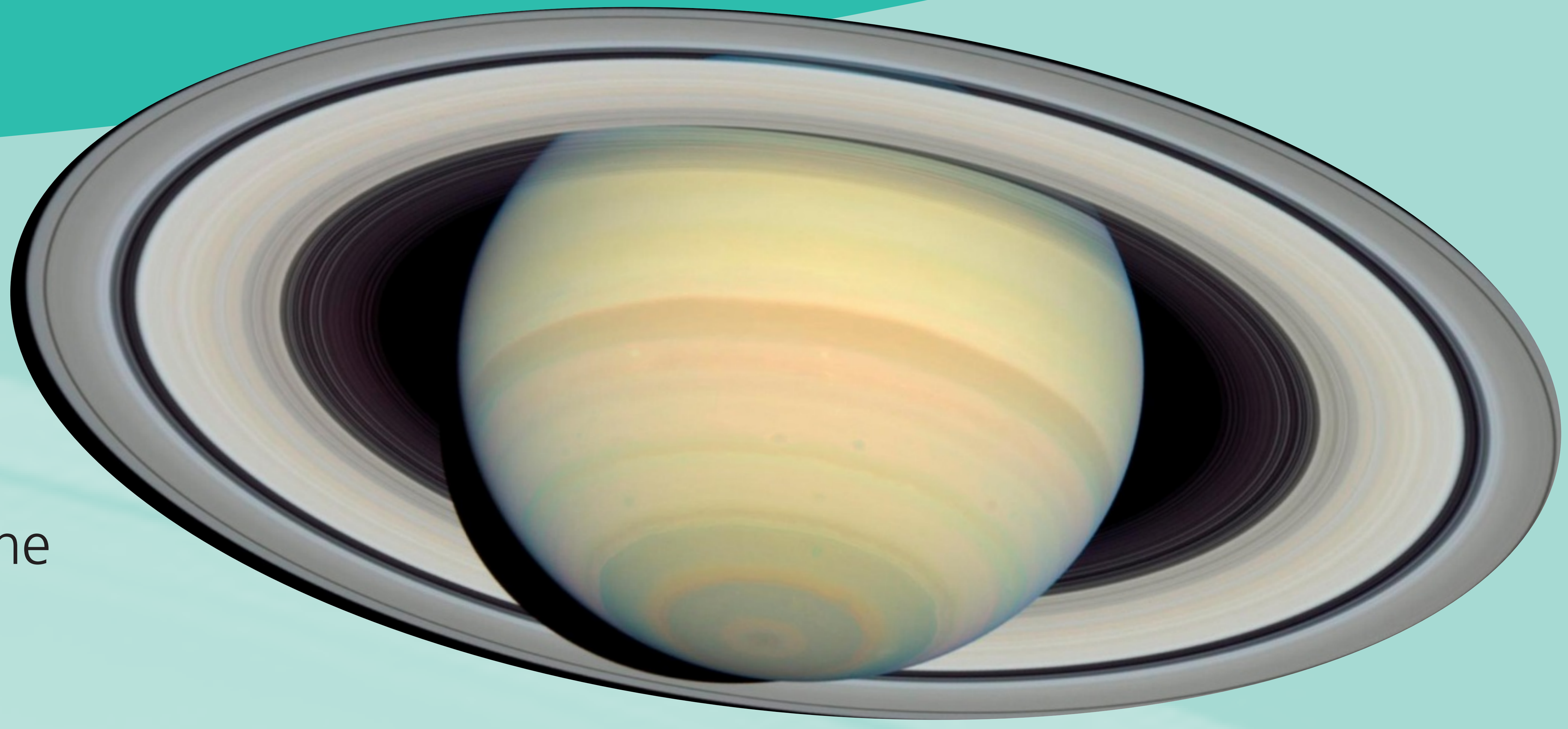

Image courtesy of NASA

The subject of the 1856 University of Cambridge Adams Prize was "*The Motion of the Rings of Saturn*".

Different hypotheses were given regarding the rings, namely that the rings were:

(i) rigid

(ii) fluid or partly gaseous, or

(iii) composed of isolated masses.

Maxwell and the other candidates were asked which of these hypotheses would be mechanically stable.

Maxwell developed equations to model Saturn and its rings. He showed that, on hypothesis (i), a uniform or non-uniform solid ring could not be stable, except for a bizarre case where a uniform solid ring had a large point mass attached to it. Observations, however, indicated that no large mass was part of the ring.

On hypothesis (ii), Maxwell showed that a fluid ring could not be stable and would break up into small portions.

Maxwell then examined a single ring composed of unconnected satellites and the propagation of waves round this ring. He extended this analysis to satellites of unequal mass, multiple rings of satellites and other issues.

Maxwell concluded that the rings were made of 'unconnected particles' - the isolated masses of hypothesis (iii). In order to appear to the eye as a solid ring, each ring would have to be made up of a very large number of independent particles.

Maxwell's essay won the 1856 Adams Prize - demonstrating his powerful ability to analyse a difficult problem mathematically. Indeed, such was the difficulty of the problem that it appears that Maxwell's essay was the *only* entry.

In 1858, Maxwell built a mechanical model, shown below, to demonstrate the wave motion of individual satellites forming a ring of Saturn.

Maxwell is rightly commemorated by having a feature of the Rings of Saturn named after him – the 'Maxwell Gap' - within the C ring.

In 2004 the NASA Cassini probe to Saturn showed that Maxwell's conclusion was right.

Maxwell's approach

For the system represented by Saturn and a solid ring, Maxwell derived the three differential equations of motion. To investigate a solid ring of variable breadth, he used Fourier analysis. By omitting higher order terms, he linearised the three equations for three unknowns. By setting the determinant to zero, he derived a biquadratic equation and obtained the conditions that would give rise to stability. As Maxwell determined that a solid ring would never satisfy these conditions, he concluded that, except for the bizarre case, a solid ring could not be stable.

Maxwell then investigated the stability of a circle of unconnected satellites. He established that such an arrangement could be stable and that four types of wave could be propagated round the ring, provided Saturn was massive enough ($S > 0{\cdot}4352\mu^2R$, where S is the mass of Saturn, R is the mass of the ring and μ is the number of satellites).

As Maxwell said in his essay:

"I have shown that such a destructive tendency (for the individual satellites to gather together under self-gravity and form one large satellite) actually exists but that by the revolution of the ring it is converted into the condition of dynamical stability ... that the only system of rings which can exist is one composed of an infinite number of unconnected particles revolving round the planet with different velocities according to their respective distances".

Reproduction of Maxwell's mechanical model of the motion of the individual particles which form the rings of Saturn, courtesy Cavendish Laboratory, Cambridge.

This panel is sponsored by
The School of Physics and Astronomy at The University of Edinburgh.

James Clerk Maxwell Foundation